Math 051 Intermediate Algebra

Lecture Notes for chapter 7.1-7.3 (audio dated 9/30/07 at 9:47 pm, 22 minutes long)

Radicals
Simplifying Radicals
Simplifying Rational Exponents

Radicals

Let us review the exponents.

Ex 1. \(5^2 = 25\)

The base is 5, power/exponent is 2, and the value is 25. That is, when base 5 is raised to the 2\(^{nd}\) power, the value is 25. We also say,

\[5 = \sqrt{25}\]

5 is the square root of 25. So, 2\(^{nd}\) power and square root are opposite (inverse) of each other.

Ex 2. \(7^3 = 343\)

The base is 7, power/exponent is 3, and the value is 343. That is, when base 7 is raised to the 3\(^{rd}\) power, the value is 343. We also say,

\[7 = \sqrt[3]{343}\]

7 is the cube root or 3\(^{rd}\) root of 343. So, 3\(^{rd}\) power and 3\(^{rd}\) root are inverse of each other.

In general, we have the nth root defined as:

\[\sqrt[n]{\text{value of the corresponding power}}\]

\[\sqrt[n]{\text{radicand}}\]

radical

Note: Find the nth root of a value is the same as finding the base.

Warm up exercises:

Ex 3. \(\sqrt{81}\) Same as: base\(^2\) = 81

Since \(9^2 = 81\), the base is 9. So, \(\sqrt{81} = 9\).
Ex 4. \[ \sqrt[3]{64} \quad \text{ Same as: base}^3 = 64 \]

Since \(4^3 = 64\), the base is 4. So, \(\sqrt[3]{64} = 4\).

Now, how to use calculator to find \(\sqrt[8]{1250}\)?

For scientific calculator users: use the \(x^y\) key. But you need to change \(\sqrt[8]{1250} = 1250^{1/8}\). We will learn the rational exponent in the next lecture. So, you key in:

\[
1250 \times \left[ \frac{1}{8} \right] = 2.4384
\]

For TI-83 users: use MATH \(\sqrt[8]{\text{key}}\).

\[
8 \text{ MATH} 5: \sqrt[8]{\text{key}} \quad 1250 \quad \text{ENTER} \quad 2.4384
\]

Let us set up a list of frequently used numbers:

\[
\begin{align*}
\sqrt{4} &= 2 \\
\sqrt{9} &= 3 \\
\sqrt{16} &= 4 \\
\sqrt{25} &= 5 \\
\sqrt{36} &= 6 \\
\sqrt{49} &= 7 \\
\sqrt{64} &= 8 \\
\sqrt{81} &= 9 \\
\sqrt{100} &= 10 \\
\sqrt[3]{8} &= 2 \\
\sqrt[3]{27} &= 3 \\
\sqrt[3]{64} &= 4 \\
\sqrt[3]{125} &= 5
\end{align*}
\]

**Simplifying Radicals**

Next we are going to look at radicals involving variables.
Ex 5. \( \sqrt[2]{16x^8y^2z^{20}} \) same as \(( )^2 = 16x^8y^2z^{20}\)

As you know, \(4^2 = 16\). So we have

\( (4) ^2 = 16x^8y^2z^{20} \)

Next using the law of exponents, \((a^m)^n = a^{mn}\). So \((x^?)^2 = x^8\). \(? = 4\). Now we have

\( (4x^4yz^{10})^2 = 16x^8y^2z^{20} \)

The base is \(4x^4yz^{10}\). That is, \(\sqrt{16x^8y^2z^{20}} = 4x^4yz^{10}\).

Ex 6. \( \sqrt[3]{125x^{12}y^{36}z^{15}} \) same as \(( )^3 = 125x^{12}y^{36}z^{15}\)

As you know, \(5^3 = 125\). Next we ask \((x^?)^3 = x^{12}\). The answer is \(x^4\) and so on.

\( \sqrt[3]{125x^{12}y^{36}z^{15}} = 5x^4y^{12}z^5 \)

Now you discover a short cut. In fact, we are using the law of exponents backward. To find the \(3^{rd}\) root of \(x^{12}\) is the same as dividing 3 into 12, i.e. index into the power.

Ex 7. \( \sqrt[3]{100} = \sqrt[12]{100} = \frac{10}{11} \)

Ex 8. \( \sqrt{0.64} = 0.8 \)

Ex 9. \( \sqrt{-16} \) not defined because base \(2 = -16\) has no solution.

Ex 10. \( -\sqrt{16} = -4 \)

Ex 11. \( \sqrt[3]{\frac{p^6}{81}} = \frac{\sqrt[3]{p^6}}{\sqrt[3]{81}} = \frac{p^2}{9} \)

Ex 12. \( \sqrt[3]{32y^{10}} = \sqrt[3]{32\sqrt[3]{y^{10}}} = 2y^{rac{10}{3}} \leftrightarrow \text{perfect square root} \)
Ex 13.  \[ \sqrt[4]{81x^4y^8z^{32}} = \sqrt[4]{81} \sqrt[4]{x^4} \sqrt[4]{y^8} \sqrt[4]{z^{32}} = 3x^1y^2z^8 \quad \text{perfect fourth root} \]

Now let us consider radicals that don’t give perfect root.

Ex 14.  \[ 3\sqrt[3]{x^{10}y^6z^{11}} \]

We will rewrite each term with two powers. That is, \( 10 = 9 + 1 \) and \( 11 = 9 + 2 \).

\[
\begin{align*}
\frac{3}{3} \sqrt[3]{10} & \quad \frac{3}{3} \sqrt[3]{11} \\
\frac{9}{1} & \quad \frac{9}{2} \\
\frac{3}{3} \sqrt[3]{10} & \quad \frac{3}{3} \sqrt[3]{11} \\
\frac{9}{1} & \quad \frac{9}{2} \\
\frac{3}{3} \sqrt[3]{10} & \quad \frac{3}{3} \sqrt[3]{11} \\
\frac{9}{1} & \quad \frac{9}{2}
\end{align*}
\]

\[ 3\sqrt[3]{x^{10}y^6z^{11}} = 3\sqrt[3]{x^9y^6z^9z^2} = 3\sqrt[3]{x^9y^6z^9xz^2} = 3\sqrt[3]{x^9y^6z^9} \sqrt[3]{xz^2} = x^3y^2z^3 \sqrt[3]{xz^2} \]

Ex 15.  \[ \sqrt[3]{108a^6b^4c^2} \]

We need to rewrite each term in the radical. 
\( 108 = 27(4) \) because \( 3^3 = 27 \)
\( a^8 = a^6 \cdot a^2 \)
\( b^4 = b^3 \cdot b \)

\[ \sqrt[3]{108a^6b^4c^2} = \sqrt[3]{27(4)a^6b^3bc^2} = \sqrt[3]{27a^6b^3} \sqrt[3]{4a^2bc^2} = 3a^2b^2 \sqrt[3]{4a^2bc^2} \]

**Simplifying Rational Exponents**

Now we understand how the nth root is related to the nth power and they are inverse of each other. So, we will define rational exponents as:

\( \sqrt[n]{x} = x^{\frac{1}{n}} \)

\( \sqrt[n]{x} = x^{\frac{1}{n}} \)

\( a^n = a^{\frac{n}{n}} \)

To simplify rational exponents, we apply the same properties of exponents.
Simplify.

Ex 1. \((x^{-\frac{1}{2}}x^{\frac{3}{4}})^6\)

We will distribute 6th power to each x-term inside the parentheses by multiplying the exponents.

\[(x^{-\frac{1}{2}}x^{\frac{3}{4}})^6 = x^{-\frac{1}{2}\cdot 6}x^{\frac{3}{4}\cdot 6} = x^{-3}x^4 = x^{4-3} = x\]

Ex 2. \((\frac{50x^{-3}y}{2xy^{-3}})^{-\frac{3}{2}}\)

It will be easier if we simplify the expression inside the parentheses first using the properties of exponents.

\[\left(\frac{50x^{-3}y}{2xy^{-3}}\right)^{-\frac{3}{2}} = \left(25x^{-3-1}y^{1-(-3)}\right)^{-\frac{3}{2}} = \left(25x^{-4}y^4\right)^{-\frac{3}{2}}\]

Now rewrite 25 as 5^2 and distribute the exponent -3/2 to each term inside the parentheses.

\[\left(\frac{50x^{-3}y}{2xy^{-3}}\right)^{-\frac{3}{2}} = \left(5^2x^{-4}y^4\right)^{-\frac{3}{2}} = 5^{2\cdot -\frac{3}{2}}x^{-4\cdot -\frac{3}{2}}y^{4\cdot -\frac{3}{2}} = 5^{-3}x^6y^{-6}\]

Now we write the terms with negative exponent as reciprocals.

\[\left(\frac{50x^{-3}y}{2xy^{-3}}\right)^{-\frac{3}{2}} = \frac{x^6}{5^3y^6} = \frac{x^6}{125y^6}\]

Ex 3. \(\left(\frac{a^{3m}b^{2m}}{c^{-5m}}\right)^{-\frac{1}{m}}\)

We will distribute the power -1/m to each term inside the parentheses and simplify.

\[\left(\frac{a^{3m}b^{2m}}{c^{-5m}}\right)^{-\frac{1}{m}} = \frac{a^{3m\cdot -\frac{1}{m}}b^{2m\cdot -\frac{1}{m}}}{c^{-5m\cdot -\frac{1}{m}}} = \frac{a^{-3}b^{-2}}{c^{5}} = \frac{1}{a^3b^2c^5}\]